Extended symmetrical classical electrodynamics

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Abstract

In the present article, we discuss a modification of classical electrodynamics in which "ordinary" point charges are absent. The modified equations contain additional terms describing the induced charges and currents. The densities of the induced charges and currents depend on the vector $\mathbf{k}$ and the vectors of the electromagnetic field $\mathbf{E}$ and $\mathbf{B}$. It is shown that the vectors $\mathbf{E}$ and $\mathbf{B}$ can be defined in terms of two 4-potentials and the components of $\mathbf{k}$ are the components of the 4-tensor of the third rank. The Lagrangian of modified electrodynamics is defined. The conditions are derived at which only one 4-potential determines the behavior of the electromagnetic field. It is also shown that static modified electrodynamics can describe the electromagnetic field in the inner region of the electric monopole. In the outer region of the electric monopole the electric field is governed by the Maxwell equations. It follows from boundary conditions at the interface between the inner and outer regions of the monopole that the vector $\mathbf{k}$ has a discrete spectrum. The electric and magnetic fields, energy and angular momentum of the monopole are found for different eigenvalues of $\mathbf{k}$.

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I. INTRODUCTION

In recent years, there has been a growing interest in the study of classical Maxwell-Chern-Simons (MCS) electrodynamics. The fundamental equations of MCS-electrodynamics are:

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{j}_o - m\mathbf{B} - \mathbf{k} \times \mathbf{E}, \tag{3}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho_o + \mathbf{k} \cdot \mathbf{B}, \tag{4}$$

where ρ_o and $\mathbf{j}_o$ are the "ordinary" charge and current densities, respectively, and "ordinary" charges are considered as point particles. The quantities m and $\mathbf{k}$ have dimensions of inverse length and ones can be considered either as the components of the 4-gradient of the dynamic pseudoscalar (axion) field [1, 2, 3, 4, 5, 6, 7] or as the components of a constant 4-pseudovector [8, 9, 10, 11, 12]. With $m = 0$ the set of the equations (1)-(4) was obtained in noncommutative electrodynamics [13].

The equations (3) and (4) can be written in the following form

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{j}_o + \frac{4\pi}{c}\mathbf{j}_i, \tag{5}$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho_o + 4\pi\rho_i, \tag{6}$$

where

$$\mathbf{j}_i = -\frac{c}{4\pi}(m\mathbf{B} + \mathbf{k} \times \mathbf{E}), \tag{7}$$

$$\rho_i = \frac{1}{4\pi}\mathbf{k} \cdot \mathbf{B}. \tag{8}$$

As distinct from the Maxwell equations, the equations (5)-(6) contain the additional quantities $\mathbf{j}_i$ and ρ_i which can be interpreted as follows [1]. The quantity $\mathbf{j}_i$ is the current density induced by magnetic and electric fields and the quantity ρ_i is the charge density induced by the magnetic field. From (7) and (8) it follows that the induced current and charge densities are the functions of the vectors of the electromagnetic field. In such a manner, an electric charge can be considered as the secondary property of the electromagnetic field. It is necessary to note that the induced current is not connected with mechanical motion of any point charges. The first term on the right-hand side of (7) has arisen in magnetohydrodynamics [8] and the second term has appeared in the description of the Hall effect [14].

In the absence of "ordinary" sources, the Maxwell equations can be considered as two sets of equations when one is transformed into the other if the replacement $\mathbf{E} \to \mathbf{B}$, $\mathbf{B} \to -\mathbf{E}$ takes place. The equations of MCS-electrodynamics do not have such symmetry. Using the equations (1)-(4) , we shall try to formulate the equations of electrodynamics which will have such symmetry. At first we assume that the equation for the curl of $\mathbf{B}$ has the same form as the equation (3) but the quantities ρ_o and $\mathbf{j}_o$ are equal to zero. Thus the equation (3) can be written in the form:

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = -m\mathbf{B} - \mathbf{k} \times \mathbf{E}. \tag{9}$$

Then we carry out the replacement $\mathbf{E} \to \mathbf{B}$, $\mathbf{B} \to -\mathbf{E}$ in the equation (9) and get the equation for the curl $\mathbf{E}$:

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -m\mathbf{E} + \mathbf{k} \times \mathbf{B}. \tag{10}$$

Suppose that the quantities m and $\mathbf{k}$ are constant in time and space. Computing the curls of the equations (9) and (10), we find the equations:

$$\Box\mathbf{B} + \frac{2}{c}\mathbf{k} \times \frac{\partial \mathbf{B}}{\partial t} + (m^2 + k^2)\mathbf{B} = \frac{2m}{c}\frac{\partial \mathbf{E}}{\partial t} - 2m\mathbf{k} \times \mathbf{E} + \nabla f_1 + \mathbf{k} f_2, \tag{11}$$

$$\Box\mathbf{E} + \frac{2}{c}\mathbf{k} \times \frac{\partial \mathbf{E}}{\partial t} + (m^2 + k^2)\mathbf{E} = 2m\mathbf{k} \times \mathbf{B} - \frac{2m}{c}\frac{\partial \mathbf{B}}{\partial t} + \nabla f_2 - \mathbf{k} f_1, \tag{12}$$

where $\Box$ is the d'Alembertian:

$$\Box = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{13}$$

and

$$f_1 = \nabla \cdot \mathbf{B} - \mathbf{k} \cdot \mathbf{E}, \tag{14}$$

$$f_2 = \nabla \cdot \mathbf{E} + \mathbf{k} \cdot \mathbf{B}. \tag{15}$$

Suppose that f_1 and f_2 are equal to zero. In this case the divergences $\mathbf{B}$ and $\mathbf{E}$ can be written as:

$$\nabla \cdot \mathbf{B} = \mathbf{k} \cdot \mathbf{E}, \tag{16}$$

$$\nabla \cdot \mathbf{E} = -\mathbf{k} \cdot \mathbf{B}. \tag{17}$$

The set of equations (9), (10),(16) and (17) can be considered as the basic set of equations of modified electrodynamics. As distinct from MCS-electrodynamics the equations (10) and (16) contain the additional terms describing the induced magnetic charges and currents.

II. THE FUNDAMENTAL EQUATIONS OF EXTENDED SYMMETRICAL CLASSICAL ELECTRODYNAMICS

Let us take the divergence of both sides of (9) and (10):

$$\frac{1}{c}\frac{\partial}{\partial t}(\nabla\cdot\mathbf{E}-\mathbf{k}\cdot\mathbf{B})-m(\nabla\cdot\mathbf{B}+\mathbf{k}\cdot\mathbf{E})=0, \tag{18}$$

$$\frac{1}{c}\frac{\partial}{\partial t}(\nabla\cdot\mathbf{B}+\mathbf{k}\cdot\mathbf{E})+m(\nabla\cdot\mathbf{E}-\mathbf{k}\cdot\mathbf{B})=0. \tag{19}$$

The equations (16) and (17) can be written in the form:

$$\nabla\cdot\mathbf{B}=4\pi\rho_m, \tag{20}$$

$$\nabla\cdot\mathbf{E}=4\pi\rho_e, \tag{21}$$

where ρ_m is the density of the induced magnetic charge:

$$\rho_m=\frac{1}{4\pi}\mathbf{k}\cdot\mathbf{E}, \tag{22}$$

and ρ_e is the density of the induced electric charge:

$$\rho_e=-\frac{1}{4\pi}\mathbf{k}\cdot\mathbf{B}. \tag{23}$$

Substituting (20)-(23) into (18) and (19), we obtain:

$$\frac{\partial\rho_e}{\partial t}-mc\rho_m=0, \tag{24}$$

$$\frac{\partial\rho_m}{\partial t}+mc\rho_e=0. \tag{25}$$

From (24) and (25) it follows that the oscillations of the densities of the induced charges must occur if $m\neq 0$. The frequency ω of such oscillations can be written as $\omega=|m|c$. The oscillations of ρ_e and ρ_m have the same magnitude but differ in phase by 90°. Consequently, an electric charge transforms into a magnetic charge and vice versa. In this case the law of charge conservation is broken and therefore we let $m=0$. With $m=0$ the equations (24) and (25) become:

$$\frac{\partial\rho_e}{\partial t}=-\frac{1}{4\pi}\mathbf{k}\cdot\frac{\partial\mathbf{B}}{\partial t}=0, \tag{26}$$

$$\frac{\partial\rho_m}{\partial t}=\frac{1}{4\pi}\mathbf{k}\cdot\frac{\partial\mathbf{E}}{\partial t}=0. \tag{27}$$

The conditions (26) and (27) mean that either the electric and magnetic fields do not depend on time or the time-varying $\mathbf{B}$ and $\mathbf{E}$ are both perpendicular to the direction of $\mathbf{k}$.

If $m = 0$ and $\mathbf{k} \neq 0$, the set of the equations (9), (10), (16) and (17) can be written

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = -\mathbf{k} \times \mathbf{E}, \tag{28}$$

$$\nabla \cdot \mathbf{E} = -\mathbf{k} \cdot \mathbf{B}, \tag{29}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = \mathbf{k} \times \mathbf{B}, \tag{30}$$

$$\nabla \cdot \mathbf{B} = \mathbf{k} \cdot \mathbf{E}. \tag{31}$$

We call the equations (28)-(31) the fundamental equations of extended symmetrical classical electrodynamics (ESC- electrodynamics). The equations (28)-(31) were postulated by authors in the article [15]. Computing the curls of the equations (28) and (30) and using (29) and (31), we find:

$$\Box \mathbf{B} + \frac{2}{c}\mathbf{k} \times \frac{\partial \mathbf{B}}{\partial t} + k^2 \mathbf{B} = 0, \tag{32}$$

$$\Box \mathbf{E} + \frac{2}{c}\mathbf{k} \times \frac{\partial \mathbf{E}}{\partial t} + k^2 \mathbf{E} = 0. \tag{33}$$

The set of the equations (28)-(31) or (32)-(33) describes the behavior of the electromagnetic field in the space region where the vector $\mathbf{k}$ is not zero. In general, the induced charge and current are distributed over this region and we call, therefore, this region the induced charge and current domain (ICC-domain). Outside the ICC-domain, the behavior of the electromagnetic field is governed by the Maxwell equations.

III. THE POTENTIALS IN ESC-ELECTRODYNAMICS

In ESC-electrodynamics we can define $\mathbf{E}$ and $\mathbf{B}$ in terms of potentials:

$$\mathbf{E} = -\nabla\phi' - \nabla \times \mathbf{A}'' - \frac{1}{c}\frac{\partial \mathbf{A}'}{\partial t} + \mathbf{k}\phi'' + \mathbf{k} \times \mathbf{A}', \tag{34}$$

$$\mathbf{B} = -\nabla\phi'' + \nabla \times \mathbf{A}' - \frac{1}{c}\frac{\partial \mathbf{A}''}{\partial t} - \mathbf{k}\phi' + \mathbf{k} \times \mathbf{A}'', \tag{35}$$

where ϕ' and ϕ'' are scalar potentials, $\mathbf{A}'$ and $\mathbf{A}''$ are vector potentials. Lorentz covariance requires that the potentials ϕ', $\mathbf{A}'$, ϕ'' and $\mathbf{A}''$ form two 4-vector potentials

$$A'^{\mu} = (\phi', \mathbf{A}'), \tag{36}$$

$$A''^{\mu} = (\phi'', \mathbf{A}''), \tag{37}$$

where we use Greek indices $\mu, \nu, ...$ to run from 0 to 3. Since $\mathbf{E}$ is a polar vector and $\mathbf{B}$ is an axial vector than the components of $\mathbf{E}$ and $\mathbf{B}$ can be interpreted as the components of the antisymmetric second rank tensor[16]:

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & -B_z & B_y \\ & & 0 & -B_x \\ & & & 0 \end{pmatrix}. \tag{38}$$

We can write the components of $F_{\mu\nu}$ as

$$F_{\mu\nu} = F'_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F''^{\rho\sigma}, \tag{39}$$

where

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu + K_{\mu\nu\alpha}A'^{\alpha}, \tag{40}$$

$$F''^{\rho\sigma} = \partial^\rho A''^{\sigma} - \partial^\sigma A''^{\rho} + K^{\rho\sigma\alpha}A''_\alpha \tag{41}$$

and $\epsilon_{\mu\nu\rho\sigma}$ is the totally antisymmetric fourth rank tensor [16]. The tensor $K_{\mu\nu\alpha}$ is a 4-tensor of the third rank and its nonvanishing components in the rest frame of the ICC-domain are

$$k_{032} = k_{203} = k_{230} = -k_{302} = -k_{023} = -k_{320} = k_x, \tag{42}$$

$$k_{013} = k_{301} = k_{310} = -k_{103} = -k_{031} = -k_{130} = k_y, \tag{43}$$

$$k_{021} = k_{102} = k_{120} = -k_{201} = -k_{012} = -k_{210} = k_z, \tag{44}$$

where the quantities k_x, k_y and k_z are the components of the vector $\mathbf{k}$. The tensor $K_{\mu\nu\alpha}$ is antisymmetric in indices μ and ν. It can be shown that

$$K_{\mu\nu\alpha}K^{\mu\nu\alpha} = 6(k_x^2 + k_y^2 + k_z^2) = 6k^2. \tag{45}$$

Thus k^2 is a Lorentz invariant. With $\mathbf{k} = 0$ the tensor (39) is the Cabibbo-Ferrari tensor [17]:

$$F_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu - \epsilon_{\mu\nu\rho\sigma}\partial^\rho A''^{\sigma}. \tag{46}$$

Let us write the equations (28) and (30) as

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{j}_e, \tag{47}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\frac{4\pi}{c}\mathbf{j}_m, \tag{48}$$

where $\mathbf{j}_e$ is the density of the induced electric current

$$\mathbf{j}_e = -\frac{c}{4\pi}\mathbf{k} \times \mathbf{E}, \tag{49}$$

and $\mathbf{j}_m$ is the density of the induced magnetic current

$$\mathbf{j}_m = -\frac{c}{4\pi}\mathbf{k} \times \mathbf{B}. \tag{50}$$

Then we introduce a 4-vector j_e^α as

$$j_e^\alpha = \frac{c}{8\pi}K^{\mu\nu\alpha}F_{\mu\nu}. \tag{51}$$

If the expressions (38) and (42)-(44) are substituted into (51), the following expression can be obtained

$$j_e^\alpha = (c\rho_e, \mathbf{j}_e), \tag{52}$$

where ρ_e and $\mathbf{j}_e$ are defined by the expressions (23) and (49), respectively. Thus the induced electric charge and current densities are the components of a 4-vector. We now introduce a 4-vector j_m^α as

$$j_m^\alpha = \frac{c}{8\pi}K^{\mu\nu\alpha}F^*_{\mu\nu}, \tag{53}$$

where $F^*_{\mu\nu}$ is the dual electromagnetic tensor

$$F^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}. \tag{54}$$

The expression (53) can be written as

$$j_m^\alpha = (c\rho_m, \mathbf{j}_m), \tag{55}$$

where ρ_m and $\mathbf{j}_m$ are defined by the expressions (23) and (50), respectively. Thus the induced magnetic charge and current densities are also the components of a 4-vector.

The Lagrangian of ESC-electrodynamics can be taken in the form

$$\mathcal{L} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} = \frac{1}{8\pi}(E^2 - B^2). \tag{56}$$

Substituting (34) and (35) into (56) and using the principle of least action [16], we obtain the basic equations (28)-(31) of ESC-electrodynamics.

IV. THE EQUATIONS FOR THE POTENTIALS

Substituting (34) and (35) into (28) and (30), we obtain :

$$\Box \mathbf{A}' + \frac{2}{c}\mathbf{k} \times \frac{\partial \mathbf{A}'}{\partial t} + k^2 \mathbf{A}' = \nabla \psi_1 + \mathbf{k}\psi_2, \tag{57}$$

$$\Box \mathbf{A}'' + \frac{2}{c}\mathbf{k} \times \frac{\partial \mathbf{A}''}{\partial t} + k^2 \mathbf{A}'' = \nabla \psi_3 - \mathbf{k}\psi_4, \tag{58}$$

$$\nabla^2 \phi' + \frac{1}{c}\frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}' + \mathbf{k} \cdot \mathbf{A}'') + k^2 \phi' = 0, \tag{59}$$

$$\nabla^2 \phi'' + \frac{1}{c}\frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}'' - \mathbf{k} \cdot \mathbf{A}') + k^2 \phi'' = 0, \tag{60}$$

where

$$\psi_1 = \nabla \cdot \mathbf{A}' + \frac{1}{c}\frac{\partial \phi'}{\partial t} - \mathbf{k} \cdot \mathbf{A}'', \tag{61}$$

$$\psi_2 = \nabla \cdot \mathbf{A}'' - \frac{1}{c}\frac{\partial \phi''}{\partial t} + \mathbf{k} \cdot \mathbf{A}', \tag{62}$$

$$\psi_3 = \nabla \cdot \mathbf{A}'' + \frac{1}{c}\frac{\partial \phi''}{\partial t} + \mathbf{k} \cdot \mathbf{A}', \tag{63}$$

$$\psi_4 = \nabla \cdot \mathbf{A}' - \frac{1}{c}\frac{\partial \phi'}{\partial t} - \mathbf{k} \cdot \mathbf{A}''. \tag{64}$$

In order to determine the divergence of $\mathbf{A}'$ and $\mathbf{A}''$, we can use the expressions (61) and (63). We assume that $\psi_1 = 0$ and $\psi_3 = 0$. In this case we have

$$\nabla \cdot \mathbf{A}' + \frac{1}{c}\frac{\partial \phi'}{\partial t} = \mathbf{k} \cdot \mathbf{A}'', \tag{65}$$

$$\nabla \cdot \mathbf{A}'' + \frac{1}{c}\frac{\partial \phi''}{\partial t} = -\mathbf{k} \cdot \mathbf{A}'. \tag{66}$$

Substituting (65) and (66) into (57)-(60), we obtain:

$$\Box \mathbf{A}' + \frac{2}{c}\mathbf{k} \times \frac{\partial \mathbf{A}'}{\partial t} + k^2 \mathbf{A}' = -\frac{2}{c}\mathbf{k}\frac{\partial \phi''}{\partial t}, \tag{67}$$

$$\Box \mathbf{A}'' + \frac{2}{c}\mathbf{k} \times \frac{\partial \mathbf{A}''}{\partial t} + k^2 \mathbf{A}'' = \frac{2}{c}\mathbf{k}\frac{\partial \phi'}{\partial t}. \tag{68}$$

$$\Box \phi' + k^2 \phi' = -\frac{2}{c}\mathbf{k} \cdot \frac{\partial \mathbf{A}''}{\partial t}, \tag{69}$$

$$\Box \phi'' + k^2 \phi'' = \frac{2}{c}\mathbf{k} \cdot \frac{\partial \mathbf{A}'}{\partial t}. \tag{70}$$

The set of the equations (65)- (70) is equivalent to the set of the equations (28)-(31).

Let us examine the case in which the electromagnetic field is defined in terms of only one 4-potential. Let $A'^{\mu} \neq 0$ and $A''^{\mu} = 0$. From the equations (65)-(70) it follows that the following conditions must be satisfied

$$\nabla \cdot \mathbf{A}' + \frac{1}{c}\frac{\partial \phi'}{\partial t} = 0, \tag{71}$$

$$\mathbf{k} \cdot \mathbf{A}' = 0, \tag{72}$$

$$\frac{\partial \phi'}{\partial t} = 0. \tag{73}$$

Substituting (73) into (71), we find

$$\nabla \cdot \mathbf{A}' = 0. \tag{74}$$

Thus the vector-potential $\mathbf{A}'$ satisfies the Coulomb gauge condition. The scalar potential ϕ' of the time-varying electromagnetic field vanishes. Consequently, the time-varying electromagnetic field is governed by the equation

$$\Box \mathbf{A}' + \frac{2}{c}\mathbf{k} \times \frac{\partial \mathbf{A}'}{\partial t} + k^2 \mathbf{A}' = 0. \tag{75}$$

From (34) and (35) it follows that the vectors of the time-varying electromagnetic field are

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}'}{\partial t} + \mathbf{k} \times \mathbf{A}', \tag{76}$$

$$\mathbf{B} = \nabla \times \mathbf{A}'. \tag{77}$$

In the static case the electromagnetic field is determined by the scalar potential ϕ' and the vector-potential $\mathbf{A}'$, which are the solutions of the equations:

$$\nabla^2 \mathbf{A}' + k^2 \mathbf{A}' = 0, \tag{78}$$

$$\nabla^2 \phi' + k^2 \phi' = 0, \tag{79}$$

The vectors of the static field can be obtained from (34) and (35), which are in this case:

$$\mathbf{E} = -\nabla \phi' + \mathbf{k} \times \mathbf{A}', \tag{80}$$

$$\mathbf{B} = \nabla \times \mathbf{A}' - \mathbf{k}\phi'. \tag{81}$$

The equation similar to (79) is used to calculate the “self-induced” electrostatic field in the paper [18]. We can find similar expressions for the 4-potential A''^{μ} if $A''^{\mu} \neq 0$ and $A'^{\mu} = 0$.

V. THE STATIC ELECTROMAGNETIC FIELD IN ESC-ELECTRODYNAMICS AND THE ELECTRIC MONOPOLE

In the static case the relations (26) and (27) are satisfied automatically and the set of the basic equations of ESC-electrodynamics becomes

$$\nabla \times \mathbf{B} = -\mathbf{k} \times \mathbf{E}, \tag{82}$$

$$\nabla \cdot \mathbf{E} = -\mathbf{k} \cdot \mathbf{B}, \tag{83}$$

$$\nabla \times \mathbf{E} = \mathbf{k} \times \mathbf{B}, \tag{84}$$

$$\nabla \cdot \mathbf{B} = \mathbf{k} \cdot \mathbf{E}. \tag{85}$$

Combining the curls of the equations (82) and (84), we obtain the equations:

$$\Delta \mathbf{B} + k^2 \mathbf{B} = 0, \tag{86}$$

$$\Delta \mathbf{E} + k^2 \mathbf{E} = 0. \tag{87}$$

If we find the magnetic field from (86) then the electric field can be calculated from (82)-(85):

$$\mathbf{E} = \frac{1}{k^2}\{\mathbf{k} \times (\nabla \times \mathbf{B}) + \mathbf{k}(\nabla \cdot \mathbf{B})\}. \tag{88}$$

We call such a state of the electromagnetic field the state of the magnetic type. We can similarly find the electric field from (87) and then the magnetic field can be calculated from (82)-(85):

$$\mathbf{B} = -\frac{1}{k^2}\{\mathbf{k} \times (\nabla \times \mathbf{E}) + \mathbf{k}(\nabla \cdot \mathbf{E})\}. \tag{89}$$

We call the electromagnetic field described by (87) and (89) the state of the electric type.

A. The electromagnetic field of the electric monopole

Let us now show that the set (82)-(85) have the solution which describes the electromagnetic field of the electric monopole. Such a state of the electromagnetic field is the state of the electric type and it is defined by the equations (87) and (89). Then we assume that the ICC-domain is the sphere of radius R which is at rest with respect to an inertial frame of reference. In this case it is convenient to use spherical coordinates (r, θ, α) where θ is the

angle between $\mathbf{k}$ and $\mathbf{r}$. We denote the unit vectors of the spherical coordinate system by $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\alpha$.

The electric field of the electric monopole has spherical symmetry. In the ICC-domain of the monopole ($r < R$) such the solution of the equation (87) can be written as

$$\mathbf{E} = \frac{C}{\sqrt{kr}} J_{\frac{3}{2}}(kr)\mathbf{e}_r, \tag{90}$$

where C is a constant, $J_{\frac{3}{2}}(kr)$ is the Bessel function. The static Maxwell equations are used to find the electromagnetic field outside the ICC-domain. In the outer region of the monopole ($r > R$) the electric field $\mathbf{E}^e$ is the solution of the Laplace equation:

$$\mathbf{E}^e = \frac{q}{r^2}\mathbf{e}_r, \tag{91}$$

where q is the electric charge of the monopole. Substituting (90) in (89) we find that the magnetic field in the inner region ($r < R$) of the monopole is given by

$$\mathbf{B} = -\frac{C}{\sqrt{kr}} J_{\frac{1}{2}}(kr)\mathbf{e}_k, \tag{92}$$

where $\mathbf{e}_k$ is an unit vector parallel to $\mathbf{k}$. The magnetic field components are:

$$B_r = -\frac{C}{\sqrt{kr}} J_{\frac{1}{2}}(kr)\cos\theta, \tag{93}$$

$$B_\theta = \frac{C}{\sqrt{kr}} J_{\frac{1}{2}}(kr)\sin\theta. \tag{94}$$

From (92)-(94) it is seen that the magnetic field in the ICC-domain is a dipole field. In the outer region ($r > R$) the magnetic dipole field is a solution of the Laplace equation and its components are

$$B_r^e = \frac{2\mu}{r^3}\cos\theta, \tag{95}$$

$$B_\theta^e = \frac{\mu}{r^3}\sin\theta, \tag{96}$$

where μ is the magnetic moment of the monopole. Let us assume that the magnetic field must be continuous at the interface between the inner and outer regions of the monopole. In our case this condition is satisfied if the magnetic field vanishes at $r \geq R$. From (93)-(94) it follows that the boundary condition for the magnetic field can be satisfied only if

$$\sin(kR) = 0. \tag{97}$$

This means that k have the eigenvalues:

$$k_n^+ = n\frac{\pi}{R}, \tag{98}$$

$$k_n^- = -n\frac{\pi}{R}, \tag{99}$$

$$n = 0, 1, 2, 3, ... \tag{100}$$

Consequently, the eigenvalues of $\mathbf{k}$ are

$$\mathbf{k}_n^+ = n\frac{\pi}{R}\mathbf{e}_k, \tag{101}$$

$$\mathbf{k}_n^- = -n\frac{\pi}{R}\mathbf{e}_k. \tag{102}$$

Thus the two eigenvalues k_n^+ and k_n^- correspond to the given value of n. The solution corresponding k_n^+ we call the positive component of the n-th mode and the solution corresponding k_n^- we call the negative component of the n-th mode. The electric and magnetic vectors of the positive or negative components of n-th mode can be written as

$$\mathbf{E}_n^\pm = \pm\frac{C_n^\pm}{\sqrt{k_n^+ r}} J_{\frac{3}{2}}(k_n^+ r)\mathbf{e}_r, \tag{103}$$

$$\mathbf{B}_n^\pm = -\frac{C_n^\pm}{\sqrt{k_n^+ r}} J_{\frac{1}{2}}(k_n^+ r)\mathbf{e}_k. \tag{104}$$

In accordance with the superposition principle the electric and magnetic fields of the n-th mode are given by

$$\mathbf{E}_n = \mathbf{E}_n^+ + \mathbf{E}_n^- = \frac{Q_n}{R^2\sqrt{k_n^+ r}} J_{\frac{3}{2}}(k_n^+ r)\mathbf{e}_r, \tag{105}$$

$$\mathbf{B}_n = \mathbf{B}_n^+ + \mathbf{B}_n^- = -\frac{G_n}{R^2\sqrt{k_n^+ r}} J_{\frac{1}{2}}(k_n^+ r)\mathbf{e}_k, \tag{106}$$

where constants

$$Q_n = (C_n^+ - C_n^-)R^2, \tag{107}$$

$$G_n = (C_n^+ + C_n^-)R^2 \tag{108}$$

have dimensions of electric or magnetic charge correspondingly. In the outer region of the monopole ($r > R$) the electric field of the n-th mode is

$$\mathbf{E}_n^e = \frac{q_n}{r^2}\mathbf{e}_r, \tag{109}$$

where q_n is the electric charge of the n-th mode of the monopole. With $r = R$ the electric field is continuous and, therefore, from (105) and (109) it follows

$$Q_n = -\pi\sqrt{\frac{\pi}{2}}n(-1)^n q_n. \tag{110}$$

Thus the electric field of the monopole have only the radial component. The dipole magnetic field is non-zero in the ICC-domain of the monopole.

In accordance with the superposition principle the vector $\mathbf{k}$ is an operator that acts on the vectors $\mathbf{E}$ and $\mathbf{B}$ as follows

$$\mathbf{k}\cdot\mathbf{F} = \sum_n(\mathbf{k}_n^+\cdot\mathbf{F}_n^+ + \mathbf{k}_n^-\cdot\mathbf{F}_n^-) = \sum_n \mathbf{k}_n^+\cdot(\mathbf{F}_n^+ - \mathbf{F}_n^-) = \sum_n \mathbf{k}_n^+\cdot\tilde{\mathbf{F}}_n, \tag{111}$$

$$\mathbf{k}\times\mathbf{F} = \sum_n(\mathbf{k}_n^+\times\mathbf{F}_n^+ + \mathbf{k}_n^-\times\mathbf{F}_n^-) = \sum_n \mathbf{k}_n^+\times(\mathbf{F}_n^+ - \mathbf{F}_n^-) = \sum_n \mathbf{k}_n^+\times\tilde{\mathbf{F}}_n, \tag{112}$$

where $\mathbf{F}$ is the vector $\mathbf{E}$ or $\mathbf{B}$ and

$$\tilde{\mathbf{F}}_n = \mathbf{F}_n^+ - \mathbf{F}_n^-. \tag{113}$$

Substituting the expressions (103) and (104) in (113), we obtain for the electric monopole

$$\tilde{\mathbf{E}}_n = \mathbf{E}_n^+ - \mathbf{E}_n^- = \frac{G_n}{R^2\sqrt{k_n^+ r}}J_{\frac{3}{2}}(k_n^+ r)\mathbf{e}_r, \tag{114}$$

$$\tilde{\mathbf{B}}_n = \mathbf{B}_n^+ - \mathbf{B}_n^- = -\frac{Q_n}{R^2\sqrt{k_n^+ r}}J_{\frac{1}{2}}(k_n^+ r)\mathbf{e}_r. \tag{115}$$

With $\mathbf{k} = 0$ the equations of ESC-electrodynamics become the Maxwell equations. Hence the operator $\mathbf{k}$ does not act on the vectors of Maxwell electrodynamics

$$\mathbf{k}\cdot\mathbf{F}_M = 0, \tag{116}$$

$$\mathbf{k}\times\mathbf{F}_M = 0, \tag{117}$$

where $\mathbf{F}_M$ is the vector of the Maxwell field. Thus the densities of induced charges and currents of the electric monopole do not depend on the Maxwell field.

B. The energy and the angular moment of the electric monopole

It can be shown that, similarly to Maxwell electrodynamics, the Poynting theorem of ESC-electrodynamics can be written as follows

$$\frac{1}{8\pi}\frac{\partial w}{\partial t} + \nabla\cdot\mathbf{S} = -\mathbf{E}\cdot\mathbf{j}_e - \mathbf{B}\cdot\mathbf{j}_m, \tag{118}$$

where w is the electromagnetic energy density:

$$w = \frac{1}{8\pi}(\mathbf{E}^2 + \mathbf{B}^2) \tag{119}$$

and $\mathbf{S}$ is the Poynting vector:

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B}. \tag{120}$$

Since the electromagnetic field of the monopole is static, the Poynting theorem can be written as

$$\nabla \cdot \mathbf{S} = -\mathbf{E} \cdot \mathbf{j}_e - \mathbf{B} \cdot \mathbf{j}_m. \tag{121}$$

Substituting (105) and (106) in (120), we obtain

$$\mathbf{S} = \sum_{n,p} \frac{cQ_n G_p}{4\pi R^4 r\sqrt{k_n^+ k_p^+}} J_{\frac{3}{2}}(k_n^+ r) J_{\frac{1}{2}}(k_p^+ r) \sin\theta \mathbf{e}_\alpha. \tag{122}$$

Since the value of the Poynting vector is independent on the angle α the left-hand side of (121) vanishes. It is seen from (49) that the vector $\mathbf{j}_e$ is perpendicular to the vector $\mathbf{E}$, hence the first term on the right-hand side of (121) is zero. Vector $\mathbf{B}$ is collinear to $\mathbf{k}$, hence $\mathbf{j}_m = 0$ and the second term on the right-hand side of (121) vanishes. We see that the Poynting theorem is valid in the inner region of the monopole. In the outer region of the monopole Poynting's theorem is also valid as $\mathbf{S}$, $\mathbf{j}_e$ and $\mathbf{j}_m$ are zero in this case.

From (119) we have that the electric energy density can be written in the form

$$w_e = \frac{\mathbf{E}^2}{8\pi} \tag{123}$$

and the electric energy of the monopole is

$$W_e = \frac{1}{8\pi}\sum_{n,p}\int_V \mathbf{E}_n \cdot \mathbf{E}_p dV \tag{124}$$

Substituting (105) in (124) and integrating over the volume V^{in} of the ICC-domain, we find that electric energy of the inner region of the monopole is

$$W_e^{in} = \sum_n (W_e^{in})_n, \tag{125}$$

where $(W_e^{in})_n$ is the electric energy of the n-th mode of the inner region of the monopole:

$$(W_e^{in})_n = \frac{Q_n^2}{2\pi^3 n^2 R} = \frac{q_n^2}{4R}. \tag{126}$$

Substituting (109) in (124) and integrating over the volume V^e of the outer region of the monopole, we find that electric energy of the external region of the monopole is

$$W_e^e = \sum_{n,p} \frac{q_n q_p}{2R} = \sum_{n,p} \frac{(-1)^{n+p} Q_n Q_p}{\pi^3 npR}. \tag{127}$$

Thus the electric energy of the n-th mode of the outer region of the monopole is

$$(W_e^e)_n = \frac{q_n^2}{2R} = \frac{Q_n^2}{\pi^3 n^2 R}. \tag{128}$$

Summing (126) and (128) we find that the total electric energy of n-th mode of the monopole is

$$(W_e)_n = \frac{3Q_n^2}{2\pi^3 n^2 R} = \frac{3q_n^2}{4R}. \tag{129}$$

The expressions (127) also contain the interaction energy between n-th and p-th modes which can be written in the form

$$(W_e^e)_{np} = \frac{q_n q_p}{R} = \frac{2(-1)^{n+p} Q_n Q_p}{\pi^3 npR}. \tag{130}$$

From (119) we have that the magnetic energy density is

$$w_m = \frac{\mathbf{B}^2}{8\pi}. \tag{131}$$

and the magnetic energy of the monopole is

$$W_m = \frac{1}{8\pi} \sum_{n,p} \int_V \mathbf{B}_n \cdot \mathbf{B}_p dV \tag{132}$$

Substituting (106) in (132) and integrating over the volume of the ICC-domain, we find that magnetic energy of the inner region of the monopole is

$$W_m^{in} = \sum_n (W_m^{in})_n, \tag{133}$$

where $(W_m^{in})_n$ is the magnetic energy of the n-th mode of the inner region of the monopole

$$(W_m^{in})_n = \frac{G_n^2}{2\pi^3 n^2 R}. \tag{134}$$

The magnetic field and the magnetic energy are zero in the outer region of the monopole.

Summing (129) and (134) we find that the total electromagnetic energy of n-th mode of the monopole is

$$W_n = \frac{3Q_n^2 + G_n^2}{2\pi^3 n^2 R} = \frac{3q_n^2}{4R} + \frac{G_n^2}{2\pi^3 n^2 R}. \tag{135}$$

In ESC-electrodynamics, the electromagnetic momentum density can be written in the standard form

$$\mathbf{p} = \frac{1}{4\pi c}\mathbf{E} \times \mathbf{B}. \tag{136}$$

Hence the angular momentum of the monopole is

$$\mathbf{J} = \frac{1}{4\pi c}\sum_{n,p}\int_V (\mathbf{r} \times (\mathbf{E}_n \times \mathbf{B}_p))dV. \tag{137}$$

Substituting (105) and (106) in (137) and integrating over the volume of the ICC-domain , we find that the angular momentum of n-th mode is

$$\mathbf{J}_n = \frac{Q_n G_n}{\pi^4 c n^3}\mathbf{e}_k. \tag{138}$$

There are the components in (137), which contain the electric field of the n-th mode and the magnetic field of the p-th mode and vice versa. We call these components the cross angular momenta of the n-th and p-th modes. In our case ones can be written in the form

$$\mathbf{J}_{np} = \frac{4(nQ_pG_n - pQ_nG_p)(-1)^{n+p}}{3\pi^4 cnp(n^2 - p^2)}\mathbf{e}_k. \tag{139}$$

Now consider a particular case. If the constants Q_n and G_n satisfy to the conditions

$$Q_n = -(-1)^n nQ, \tag{140}$$

$$G_n = -(-1)^n n^2 G, \tag{141}$$

then the angular momenta (138) and (139) become

$$\mathbf{J}_n = \frac{QG}{\pi^4 c}\mathbf{e}_k, \tag{142}$$

$$\mathbf{J}_{np} = \frac{4QG}{3\pi^4 c}\mathbf{e}_k. \tag{143}$$

Substituting (140) in (110), we have

$$q_n = q = \frac{1}{\pi}\sqrt{\frac{2}{\pi}}Q. \tag{144}$$

Suppose that there is only one mode of the electromagnetic field in the ICC-domain. It follows from the expressions (142) and (144) that the angular momentum and charge of the monopole do not depend on the state number n. From (142) we see that the projection of the angular momentum of n-th state on the direction of $\mathbf{k}$ is

$$J = \frac{QG}{\pi^4 c}, \tag{145}$$

Then from (144) and (145) we have that the expression (135) can be written in the form

$$W_n = \frac{3q^2}{4R} + \frac{(\pi c n J)^2}{q^2 R}. \tag{146}$$

If J is constant then the energy minimum of the n-th state is reached at a certain magnitude of the electric charge. For example the energy of the first state of the monopole is minimum if the electric charge is

$$q' = \pm\sqrt{\frac{2\pi c|J|}{\sqrt{3}}}. \tag{147}$$

In our case electric charge of any state is equal to the electric charge of the first state and therefore the energy of any state of the monopole is

$$W_n = \frac{\sqrt{3}\pi c|J|}{2R}(1+n^2). \tag{148}$$

With $q = q'$ and $n = 1$ the electric energy of the monopole is equal to its magnetic energy. The electric monopole energy does not depend on the state number n. Thus we showed that the set of the equations (82)-(85) has the particular solutions which describe electric monopoles. From the equations (86) and (88) we can obtain the solution which describes magnetic monopoles. In general, the ICC-domain can have any multipole moment.

VI. CONCLUSIONS

In the present article the governed equations of ESC-electrodynamics are formulated in which "ordinary" point charges are absent. The equations of ESC-electrodynamics contain the additional terms describing the induced charges and currents. The densities of the induced charges and currents depend on the vector **k** and the vectors of the electromagnetic field **E** and **B**.

In ESC-electrodynamics the vectors **E** and **B** can be defined in terms of two 4-potentials. It is shown that the components of **k** are the components of the 4-tensor of the third rank. The Lagrangian is found for ESC-electrodynamics. We obtain the conditions for which there is only one 4-potential describing the behavior of the electromagnetic field.

It is shown that static ESC-electrodynamics can describe the electromagnetic field in the inner region of the electric monopole. There are both the electric and magnetic fields in the inner region. In the outer region of the electric monopole the electric field is governed by the

Maxwell equations. From the boundary conditions at the interface between the inner and outer regions it follows that the vector **k** has a discrete spectrum. The electric and magnetic fields, energy and angular momentum are found for different values of **k**. The structure of the electromagnetic field in the inner region of the monopole is changed in a discrete way when a transition occurs between different states.

There is a particular case in which different states of the monopole have the same angular momenta and charges. Such properties of the electric monopole are similar to properties of elementary particles.

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